Investments

Instructions: Answer every question. This is an open-note (and note only) examination. Total: 100 points. Time limit: 3 hours.

1. (40 points, five points each) Let $\mu_A$ and $\mu_B$ be the expected returns on two assets $A$ and $B$. Their variances are $\sigma_A^2$ and $\sigma_B^2$, respectively. If $\mu_A > \mu_B$ and $\sigma_A^2 < \sigma_B^2$. Answer the following questions. Brief explanations in three or four sentences, please.

(a) As asset $B$ is dominated by asset $A$, a risk averse investor must prefer asset $A$ to asset $B$, and asset $B$ plays no role in the market.

(b) Asset $A$ is more efficient than asset $B$ in terms of the “mean-variance” framework.

(c) If the market is in equilibrium, this implies that $\beta_A > \beta_B$, regardless of the efficiency of the two assets.

(d) Asset $A$ will take a larger weight than asset $B$ in the market portfolio, if both assets are uncorrelated with the rest of the market.

(e) Consider an optimal portfolio composed of these two assets and a riskless asset. Asset $B$ will have a non-zero weight in the portfolio only when there is no short-sale restriction on all assets.

(f) If two assets have the same expected return, then it is not possible to construct a portfolio that gives a higher expected return.

(g) You can construct a portfolio with beta of 0.8 by investing 80 percent of the investment budget in the market portfolio and the remainder in T-bills.

(h) If the market is in equilibrium, only asset $A$ is possible to lie above the security market line, not asset $B$.

2. Optimal portfolio consisting of two assets.

(a) (10 points) Derive the weights for an optimal weight composed of two risky assets, with appropriate expected returns and variances and covariance.

(b) (10 points) Suppose now you know that

$$r_i = a_i + b_i r_m + e_i, i = 1, 2,$$

where, $a_i$ and $b_i$ are known; $r_m$ has mean $\mu_m$ and variance $\sigma_m^2$, and $cov(e_1, e_2) = 0$. Express the optimal weights in terms of the parameters.
3. (20 points) Two stocks are priced differently as the following.

<table>
<thead>
<tr>
<th>month</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Stock B</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Calculate the following:

(a) Calculate the three-month cumulative return of an equal-weighted rebalanced portfolio consisting of these two stocks.

(b) Calculate the three-month cumulative return of an equal-weighted buy-and-hold portfolio consisting of these two stocks.

(c) Calculate the three-month cumulative return of a price-weighted buy-and-hold portfolio consisting of these two stocks.

(d) Calculate the annualized average monthly return of the above price-weighted portfolio.

4. (10 points) Two investment advisers are comparing performance. One averaged a 19% rate of return and the other a 16% rate of return. However, the beta of the first adviser was 1.5, whereas that of the second was 1. If the T-bill rate were 6% and the market return during the period were 14%, which investor would be the superior stock selector?

5. (10 points) Brief explanation. The CAPM says that in equilibrium, the market portfolio, which is mean-variance efficient, will be a portfolio of all assets, with the weight of each asset proportional to its market value. But when you estimate the optimal weights with the sample average returns and sample covariance matrix using historical data, you sometimes get negative weights. A contradiction? Why?