1. (25 points) Let $\mu_A$ and $\mu_B$ be the expected returns on two assets $A$ and $B$. Their variances are $\sigma^2_A$ and $\sigma^2_B$, respectively. Also, $\mu_A > \mu_B$ and $\sigma^2_A < \sigma^2_B$.

(a) (10 points) Answer the following questions. Brief explanations in three or four sentences, please.
   i. In equilibrium, both assets lie on the SML, but not on the CML, implying that both are not mean-variance efficient.
   ii. If the market is in equilibrium, asset A has a larger market value than asset B.

(b) (10 points) Suppose the two assets are correlated with covariance $\sigma_{ab}$. Let $W$ denote the proportion of the optimal portfolio invested in asset A. Derive $\frac{\partial W}{\partial \sigma_{ab}}$. You may also replace the covariance with the correlation coefficient $\rho$. If you cannot derive the formula, please explain in words the relation between $W$ and the covariance.

(c) (5 points) In the absence of a riskless asset, investors may no longer hold the market portfolio. However, every investor’s portfolio can still be represented as a weighted average of the market portfolio and the zero-beta portfolio. Comment, please.

2. (15 points) On returns.

(a) (5 points) Suppose a bank offers an annual return of 12%. You deposit $100 in the beginning of a year, what would be your final balance if your saving is compounded continuously?

(b) (5 points) Continue the above question. What if it is compounded monthly?

(c) (5 points) Two banks are competing by offering different interest rates. Bank A offers an annual simple return of 12%, whereas Bank B offers a continuously compounded annual return of 9.531%. Which one is better if you were to deposit your money for two years?

3. (25 points) Suppose you perform regression of raw returns on market index returns, and get the following result.

<table>
<thead>
<tr>
<th></th>
<th>alpha</th>
<th>beta</th>
<th>$\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>-0.4</td>
<td>0.8</td>
<td>25</td>
</tr>
<tr>
<td>Stock B</td>
<td>0.4</td>
<td>1.2</td>
<td>20</td>
</tr>
</tbody>
</table>
Let $\bar{R}_m = 10\%$, $\sigma_m = 20\%$.

(a) What is the risk free rate, if the CAPM holds?

(b) What is $corr(R_A, R_B)$, according to the “single index market model”?

(c) Consider a minimum variance portfolio composed of these two assets, what is the weight for asset A?

(d) Calculate the weight for stock A for an optimal portfolio maximizing the Sharpe measure.

(e) Now, suppose you think that next year the average market return will drop from 10% to 5%, while leaving its volatility (i.e., $\sigma_m$) unchanged. What would be the optimal composition?

4. (20 points) Two stocks are priced differently as the following.

<table>
<thead>
<tr>
<th>month</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Stock B</td>
<td>30</td>
<td>15</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Calculate the following:

(a) Calculate the average monthly return of an equal-weighted rebalanced portfolio consisting of these two stocks.

(b) Suppose initially you implement the above equal-weighted portfolio by investing $200 at month 0. How many shares of stocks you have at time 3?

(c) Calculate the three-month cumulative return of an equal-weighted buy-and-hold portfolio consisting of these two stocks.

(d) Calculate the annualized return of a price-weighted buy-and-hold portfolio consisting of these two stocks holding from month 0 to month 3.

5. (15 points) Suppose we know the correlation coefficients for 4 different assets: $\text{Corr}(A, B) = 0.8$; $\text{Corr}(A, C) = 0.60$; $\text{Corr}(A, D) = 0.4$. Each stock has an expected return of 5% and a standard deviation of 16%.

(a) (5 points) If your current portfolio is composed of Stock A only, and now you can add some of only one stock to your portfolio, which stock would you choose? Explain in detail.

(b) (5 points) Would the answer to the above problem change for more risk-averse investors?

(c) (5 points) Suppose that in addition to investing in one more stock, you can invest in T-bills as well. Would you change your answers to problem (a) if the T-bill rate is 8%?